Ultrasound Image Reconstruction by Solving an Inverse Problem with Denoising Diffusion Restoration Models

Yuxin Zhang, Clément Huneau, Jérôme Idier, Diana Mateus

yuxin.zhang@ls2n.fr

Problem

Medical ultrasound images can be reconstructed by solving an inverse problem $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{x} \in \Re^{N\times 1}$ contains the ultrasonic reflectivity values at N pixel positions, $\mathbf{y} \in \Re^{K\times 1}$ gathers the sampled channel data, $\mathbf{H} \in \Re^{K\times N}$ is the model matrix containing the information of geometry and the point-spread function, and $\mathbf{n} \in \Re^{K\times 1}$ stands for the white Gaussian noise with the standard deviation σ. The inverse problem was traditionally solved with one or several regularization functions (s.t. $l1$ norm, $l2$ norm, wavelet-based term), but with this approach, it is hard to get a satisfied balance between the image contrast, the spatial resolution, and the speckle preservation which are important features for medical ultrasound images.

Basic Concepts

Denoising Diffusion Restoration Models (DDRM)[2] were recently proposed to solve inverse problems by leveraging the pre-trained Denoising Diffusion Probabilistic Models (DDPM)[3] which are generative models unconditioned on any inverse problem.

With the SVD of the model matrix $\mathbf{H}_d$, an inverse problem can be transformed into a denoising problem (Equ.1 to Equ.3)

$$\mathbf{y}_d = \mathbf{H}_d\mathbf{x}_d + \mathbf{n}_d \tag{1}$$

$$\mathbf{y}_d = \mathbf{U}_d\Sigma_d\mathbf{V}_d^t\mathbf{x}_d + \mathbf{n}_d \tag{2}$$

$$\bar{\mathbf{y}}_d = \bar{\mathbf{x}}_d + \bar{\mathbf{n}}_d, \tag{3}$$

where $\bar{\mathbf{y}}_d = \Sigma_d^\dagger\mathbf{U}_d^t\mathbf{y}_d$, $\bar{\mathbf{x}}_d = \mathbf{V}_d^t\mathbf{x}_d$, and $\bar{\mathbf{n}}_d = \Sigma_d^\dagger\mathbf{U}_d^t\mathbf{n}_d$.

The inverse problem conditioned diffusion process is realized by weighting the trust for $\bar{\mathbf{y}}_d$ and DDPM according to the standard deviation of the additive noise $\sigma_d = std(\mathbf{n}_d)$.

After certain time steps, an estimated denoised $\bar{\mathbf{x}}_d$ can be generated and the estimated ground truth $\mathbf{x}_d$ can then be returned by calculating $\mathbf{V}_d\bar{\mathbf{x}}_d$.

Models

The traditional inverse problem of ultrasound beamforming has the size of $\mathbf{y}$ always larger than that of $\mathbf{x}$. To compress the amount of data, we use the matched filtering to transform the inverse problem to

$$\mathbf{H}^t\mathbf{y} = \mathbf{H}^t\mathbf{H}\mathbf{x} + \mathbf{H}^t\mathbf{n} \tag{4}$$

However, DDRM assumes that the additive noise is *i.i.d* Gaussian, which is not satisfied when the noise is colored by $\mathbf{H}^t$. Therefore, we consider a whitening operator $\mathbf{C} \in \Re^{N\times N}$ to further transform the inverse problem to

$$\mathbf{C}\mathbf{H}^t\mathbf{y} = \mathbf{C}\mathbf{H}^t\mathbf{H}\mathbf{x} + \mathbf{C}\mathbf{H}^t\mathbf{n}. \tag{5}$$

In our experiments, we perform DDRM with both inverse problems.

C and $svd(H_d)$

Defining the diagonal matrix $\Lambda \in \Re^{N\times N}$ and the full matrix $\mathbf{V} \in \Re^{N\times N}$ as the eigenvalues and the right eigenvectors of $\mathbf{H}^t\mathbf{H}$ respectively, and the whitening operator can be calculated as

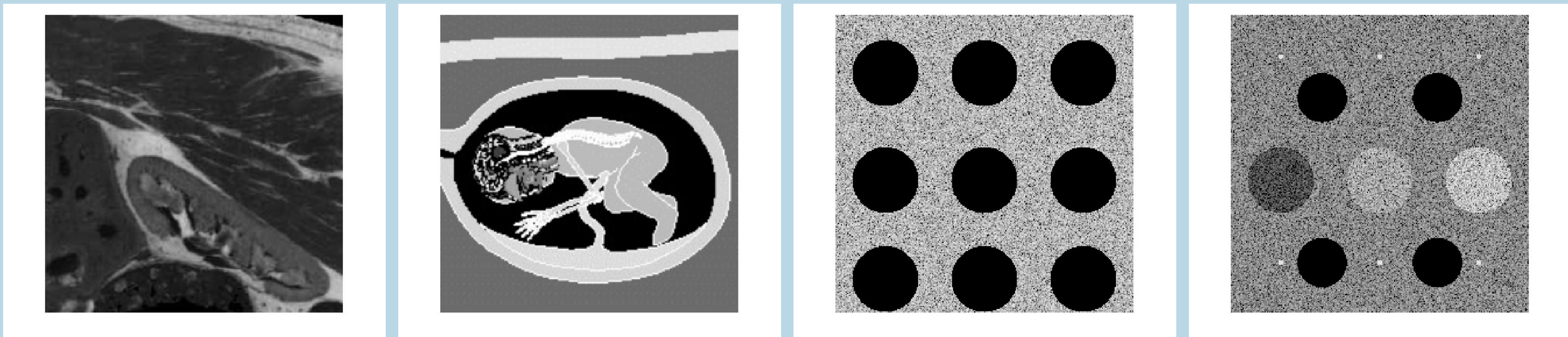
$$\mathbf{C} = \Lambda^{\frac{1}{2}}\mathbf{V}^t,$$

When DDRM is applied with Equ.4,
$svd(\mathbf{H}_d) = svd(\mathbf{H}^t\mathbf{H}) = \mathbf{V}\Lambda\mathbf{V}^t$, $\sigma_d = std(\mathbf{H}^t\mathbf{n})$;

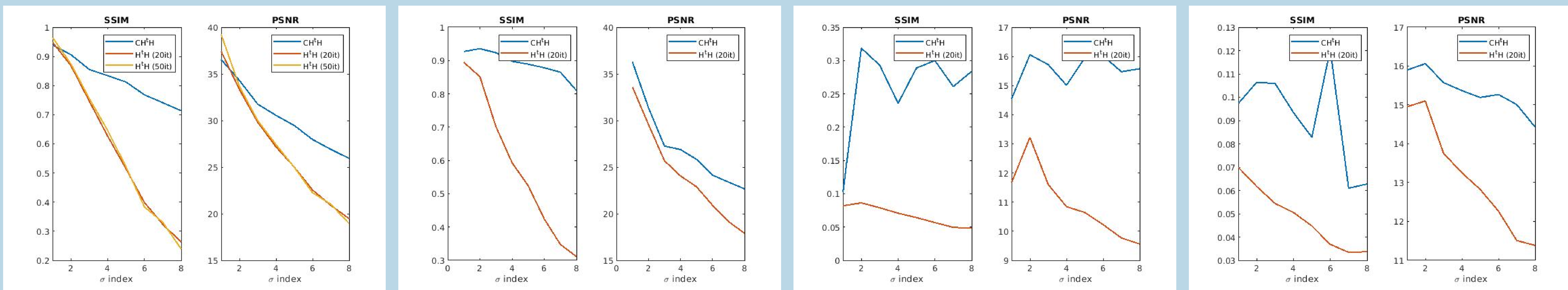
when DDRM is applied with Equ.5,
$svd(\mathbf{H}_d) = svd(\mathbf{C}\mathbf{H}^t\mathbf{H}) = \mathbf{I}\Lambda^{\frac{1}{2}}\mathbf{V}^t$, $\sigma_d = \sigma(\mathbf{n})$;

Simulation Results

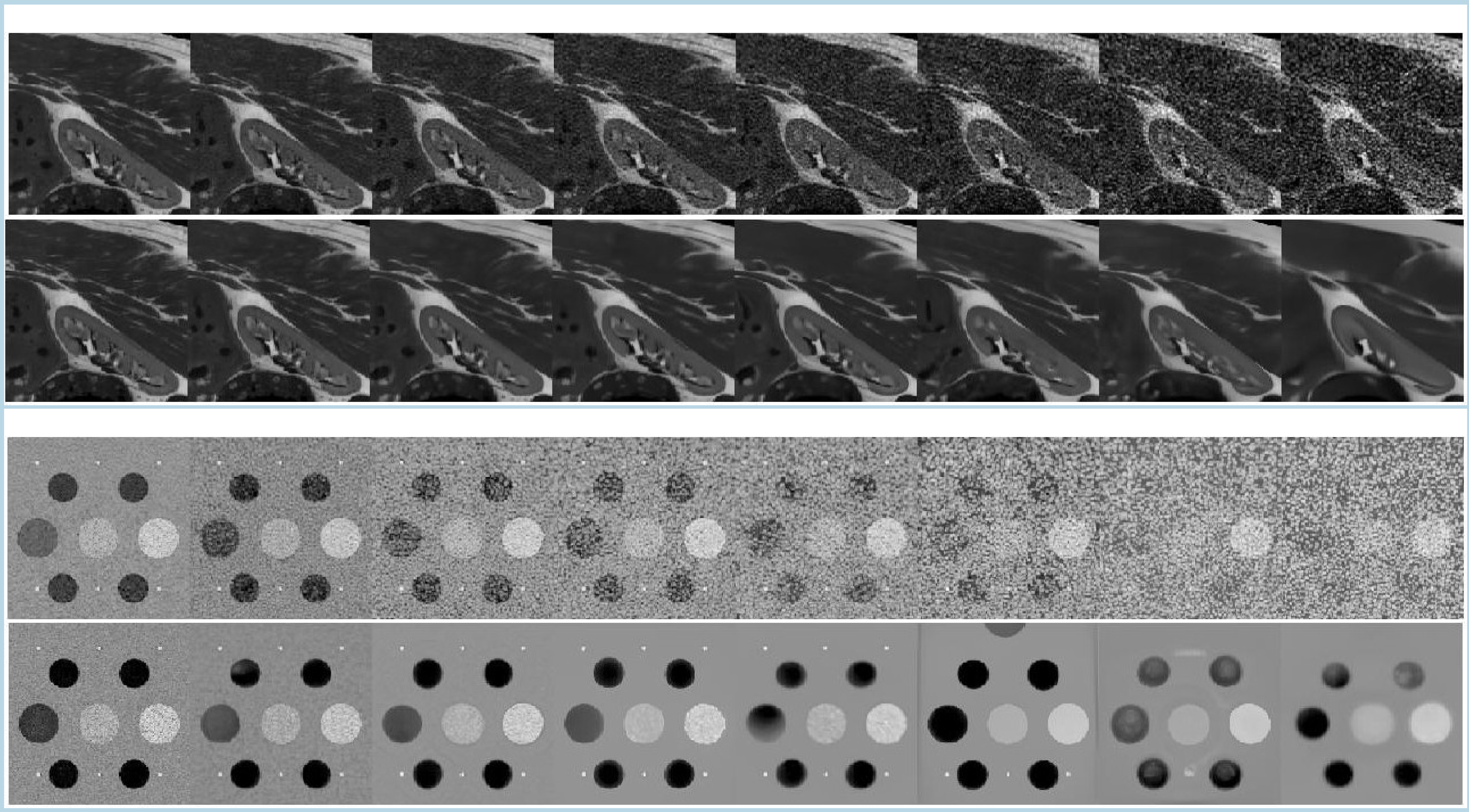
We test 4 different phantoms for checking the adaptation of our inverse problem models to DDRM, and the ground truth images of the phantoms (kidney, fetus[1], simu1, and simu2 from left to right) are



The two inverse problem models are applied with varying the standard deviation of the additive noise $\mathbf{n}$ from small to large, and the performances are evaluated with the SSIM and PSNR of the restored images.



The visualized comparisons of the two models' performances based on the kidney phantom and the simu2 phantom are below, where the standard deviation of the additive noise increases from left to right. The upper row and the lower row for each phantom case correspond to the results with Equ.4 and with Equ.5 respectively.



Conclusion

Our upgraded inverse problem model with a whitening operator adapts well to DDRM. And the effect of the whitening operator becomes more obvious with high-level additive noise.

The value range of the simulated ground truth used above is $[0, 1]$ where the values of the darkest pixels are the smallest, but it is not the case for ultrasound images where the range is $[-1, 1]$. Our future work will be on this data domain maladaptation problem.

References

[1] Field II Ultrasound Simulation Program.

[2] Bahjat Kawar, Michael Elad, Stefano Ermon, and Jiaming Song. Denoising Diffusion Restoration Models, February 2022. arXiv:2201.11793 [cs, eess].

[3] Alex Nichol and Prafulla Dhariwal. Improved Denoising Diffusion Probabilistic Models, February 2021. arXiv:2102.09672 [cs, stat].