

# Ultrasound Image Reconstruction with Denoising Diffusion Restoration Models

## DGM4MICCAI - 2023

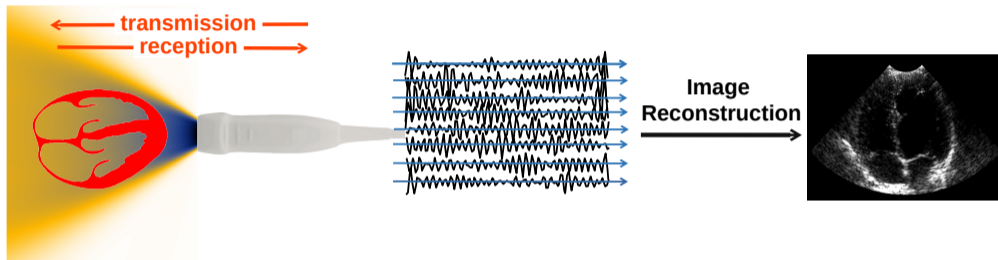
Yuxin Zhang

**Supervisors** : Clément Huneau, Jérôme Idier, Diana Mateus

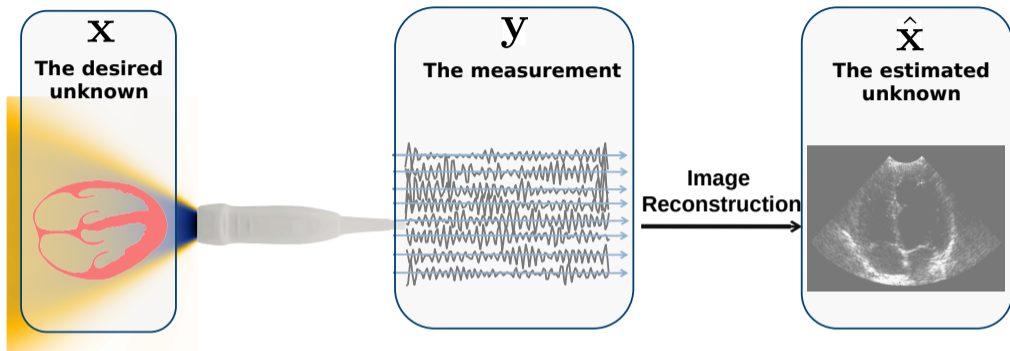
Nantes Université, École Centrale Nantes, LS2N,  
CNRS, UMR 6004, F-44000 Nantes, France

8 - October - 2023



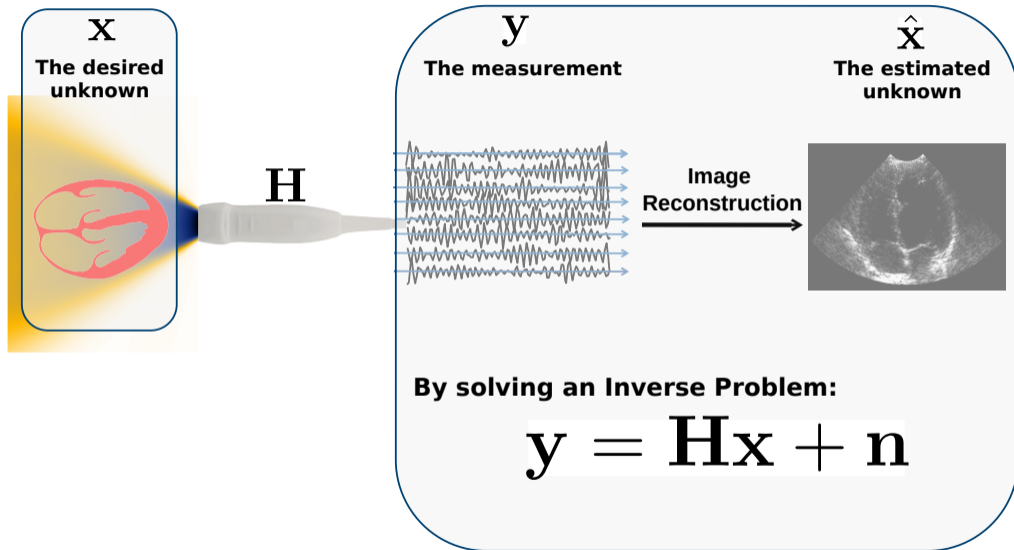


Source : [https://www.biomecardio.com/files/Tracking\\_motions\\_in\\_the\\_body.pdf](https://www.biomecardio.com/files/Tracking_motions_in_the_body.pdf)



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# Image Reconstruction $\rightarrow$ an Inverse Problem



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## Model-based

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \phi_{\text{reg}}$$

Ozkan et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2018

Goudarzi et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2022



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## Learning-based



Perdios et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. (accepted)



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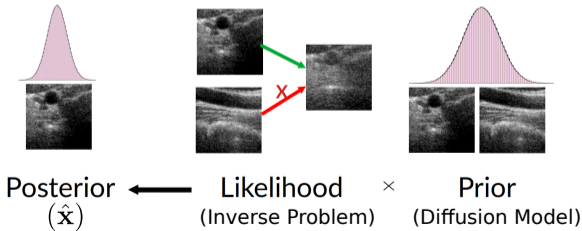
## Learning-based



Perdios et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. (accepted)

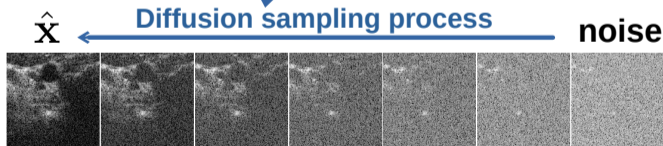


**Hybrid**



$$y = \mathbf{H}x + \mathbf{n}$$

- Song Y et al. Solving inverse problems in medical imaging with score-based generative models. ICLR, 2022
- Song J et al. Pseudoinverse-guided diffusion models for inverse problems. ICLR, 2023
- Chung H et al. Score-based diffusion models for accelerated MRI. Med Image Anal. 2022
- Chung H et al. Diffusion posterior sampling for general noisy inverse problems. ICLR, 2023
- Kavar B et al. Denoising diffusion restoration models. NeurIPS. 2022 (**DDRM**)





$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

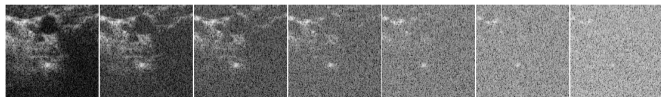
$$\Sigma^\dagger \mathbf{U}^T \mathbf{y} = \mathbf{V}^T \mathbf{x} + \Sigma^\dagger \mathbf{U}^T \mathbf{n}$$

$$\bar{\mathbf{y}} = \bar{\mathbf{x}} + \bar{\mathbf{n}}$$

 $\hat{\mathbf{x}}$ 

Diffusion sampling process

noise



**DDRM** runs “denoising and/or inpainting” in the space transformed by **Singular Value Decomposition**

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$$

**DDRM** (Kawar et al. NeurIPS 2022)  
initially for **natural images**

$$\mathbf{y} = \text{SVD}(\mathbf{H}) \begin{matrix} \boxed{\text{grid}} & \boxed{\text{dots}} & \boxed{\text{grid}} \end{matrix} \mathbf{x} + \text{white } \mathbf{n}$$

$$\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{B}\mathbf{H}) \begin{matrix} \boxed{\text{grid}} & \boxed{\text{dots}} & \boxed{\text{grid}} \end{matrix} \mathbf{x} + \text{colored } \mathbf{B}\mathbf{n}$$

$$\mathbf{C}\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{C}\mathbf{B}\mathbf{H}) \begin{matrix} \boxed{\text{grid}} & \boxed{\text{dots}} & \boxed{\text{grid}} \end{matrix} \mathbf{x} + \text{white } \mathbf{C}\mathbf{B}\mathbf{n}$$

data compressing



noise whitening



$$\mathbf{y} = \text{SVD}(\mathbf{H}) \begin{matrix} \text{[grid]} & \text{[dots]} & \text{[grid]} \end{matrix} \mathbf{x} + \text{white } \mathbf{n}$$

$$\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{B}\mathbf{H}) \begin{matrix} \text{[grid]} & \text{[dots]} & \text{[grid]} \end{matrix} \mathbf{x} + \text{colored } \mathbf{B}\mathbf{n}$$

$$\mathbf{C}\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{C}\mathbf{B}\mathbf{H}) \begin{matrix} \text{[grid]} & \text{[dots]} & \text{[grid]} \end{matrix} \mathbf{x} + \text{white } \mathbf{C}\mathbf{B}\mathbf{n}$$

data compressing

noise whitening



## Natural Images

VS

## Ultrasound Images (SIGNED)

Pre-trained on :



Figure – the ImageNet dataset (1,281,167 images) (?)

Fine-tuned on :

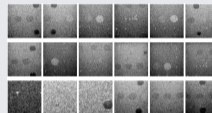


Figure – Examples of the self-acquired dataset (800 images)

Test set : PICMUS dataset (?) gives the observation  $y$ .

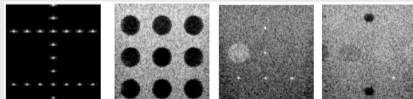
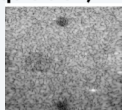
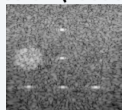
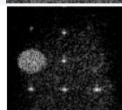


Figure – Examples of PICMUS reconstructed ultrasound images

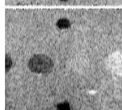
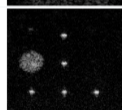
## 1 transmission (Fast acquisition)



Baseline (DAS1)

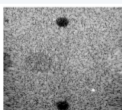
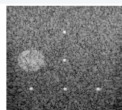


**DRUS (ours)**  
 $(\mathbf{B}_y = \mathbf{B}H_x + \mathbf{B}_n)$



**WDRUS (ours)**  
 $(\mathbf{CB}_y = \mathbf{CB}H_x + \mathbf{CB}_n)$

## 75 transmissions (Slow acquisition)



Golden standard  
 (DAS75)

	Resolution (FWHM [mm]↓)		Contrast (CNR[dB] ↑)
	Axial	Lateral	
Baseline	0.51	1.21	8.15
<b>DRUS</b>	0.26	0.69	<b>12.9</b>
<b>WDRUS</b>	<b>0.25</b>	0.62	11.95
Golden standard	0.49	0.59	12.05



**Diffusion Inverse Problem Solver**

**Model-based**

**Learning-based**



**Ultrasound Inverse Problem Model**

**noise whitened**

**data compressed**

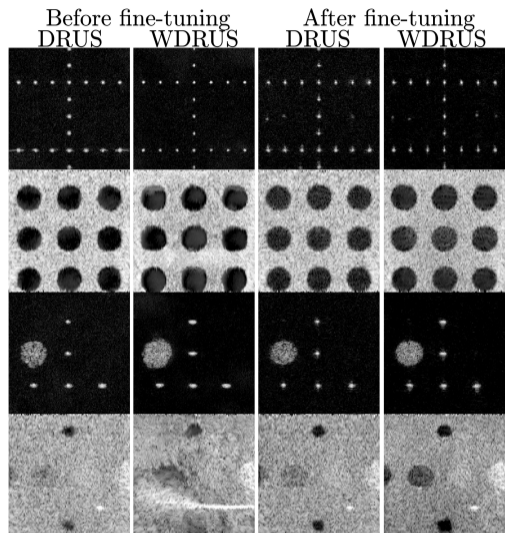
**original**

```
graph LR; original --> data_compressed; data_compressed --> noise_whitened;
```



**Fine-Tuning from a Natural-Image Diffusion Model**

Thank you !





# B and C in a simple case

$$*\mathbf{B} = \mathbf{H}^t$$

$$*\mathbf{C} = \mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{V}^t, \text{ where } \text{eig}(\mathbf{B}\mathbf{B}^t) = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^t$$

$$\text{Cov}(\mathbf{C}\mathbf{B}\mathbf{n}) = \text{E} [\mathbf{C}\mathbf{B}\mathbf{n}\mathbf{n}^t\mathbf{B}^t\mathbf{C}^t] = \gamma^2 \mathbf{C}\mathbf{B}\mathbf{B}^t\mathbf{C}^t = \gamma^2 \mathbf{C}\mathbf{V}\mathbf{\Lambda}\mathbf{V}^t\mathbf{C}^t = \gamma^2 \mathbf{I}_M$$

## In summary

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\underline{\quad} \mathbf{B}\mathbf{y} = \mathbf{B}\mathbf{H}\mathbf{x} + \mathbf{B}\mathbf{n} \text{ (DRUS)}$$

$$\underline{\quad} \mathbf{C}\mathbf{B}\mathbf{y} = \mathbf{C}\mathbf{B}\mathbf{H}\mathbf{x} + \mathbf{C}\mathbf{B}\mathbf{n} \text{ (WDRUS)}$$

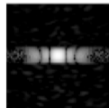
ground truth (x)



measurement (y)



$\mathbf{B}\mathbf{y}$



$\mathbf{C}\mathbf{B}\mathbf{y}$

