Ultrasound Imaging based on the Variance of a Diffusion Restoration Model

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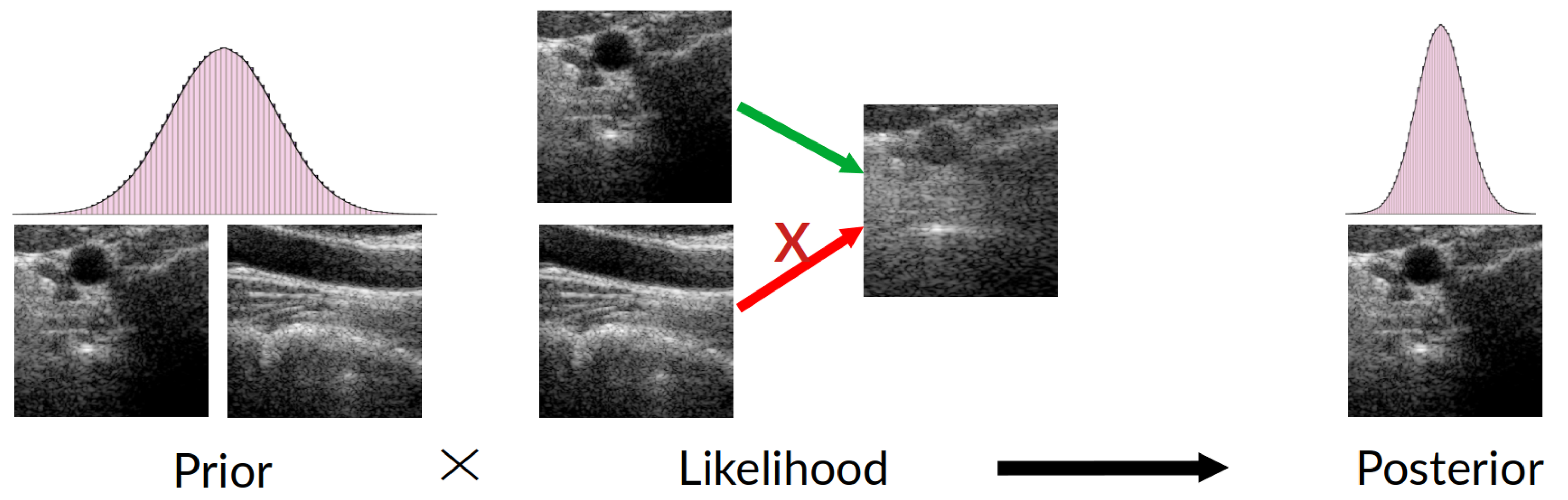
1/ Background

Ultrasound (US) imaging is valued in medical diagnostics for its real-time, affordability, portability, and minimal invasiveness. However, conventional algorithms often yield suboptimal image quality in terms of SNR, contrast, and spatial resolution. Recently, there has been progress in both model-based and learning- based approaches addressing the problem of ultrasound image reconstruction. Bringing the best from both worlds, We propose a hybrid model-based deep learning solution that incorporates a physical model and a learned diffusion model and thus, does not require retraining for a new task [1].



2/ Ultrasound imaging inverse problem

The traditional inverse problem model of plane-wave US imaging is :

$$\underbrace{\mathbf{y}_i^{K\times 1}}_{\text{measured signals}} = \underbrace{\mathbf{H}_i^{K\times N}}_{\text{model matrix}}\mathbf{x}^{N\times 1} + \underbrace{\mathbf{n}_i^{K\times 1}}_{\text{additive noise}}, \quad \text{where } \mathbf{x} = \underbrace{\mathbf{m}^{N\times 1}}_{\text{reflectivity map}} \odot \underbrace{\mathbf{p}^{N\times 1}}_{\text{echogenicity map}}, \quad K > N = 256^2$$

(multiplicative noise)

To compress the huge matrix $\mathbf{H}$, we project the measurements to the image domain by using a weighted matched filter matrix $\mathbf{B} \in \mathbb{R}^{N\times K}$:

$$\mathbf{B}_i\mathbf{y}_i = \mathbf{B}_i\mathbf{H}_i\mathbf{x} + \mathbf{B}_i\mathbf{n}_i \qquad (1)$$

3/ Diffusion Reconstruction of US images

We proposed **DRUS** (Diffusion Reconstruction of US images[1]) employing DDRM (Denoising Diffusion Restoration Models[2]) to estimate $\mathbf{x}$ from a single PW based on (1).

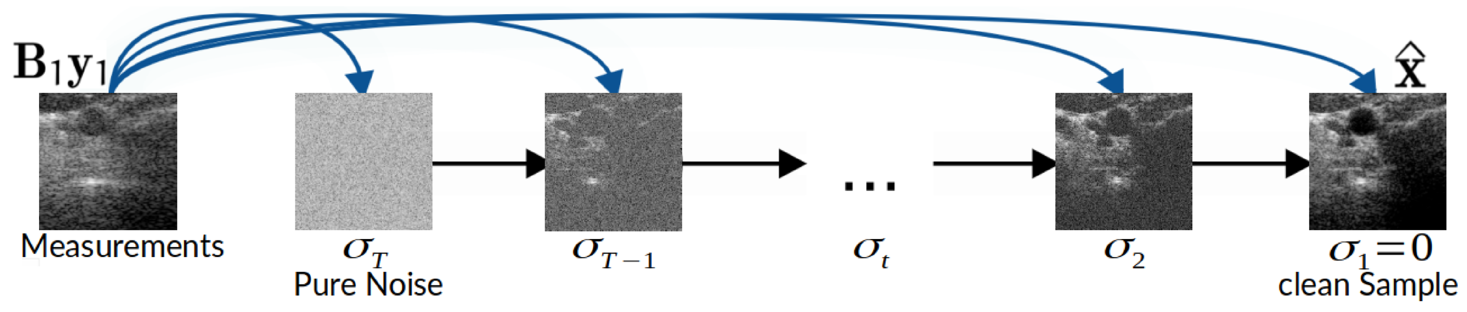
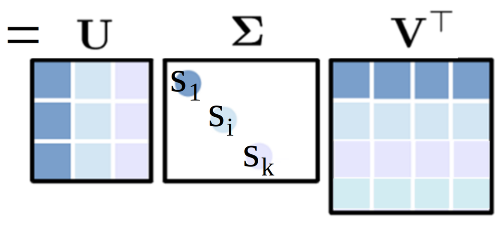


Figure 1 – Conditional diffusion sampling.

DDRM runs "denoising" in the space transformed by $\mathrm{svd}(\mathbf{BH})$.

$$\mathbf{BH} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^\top$$

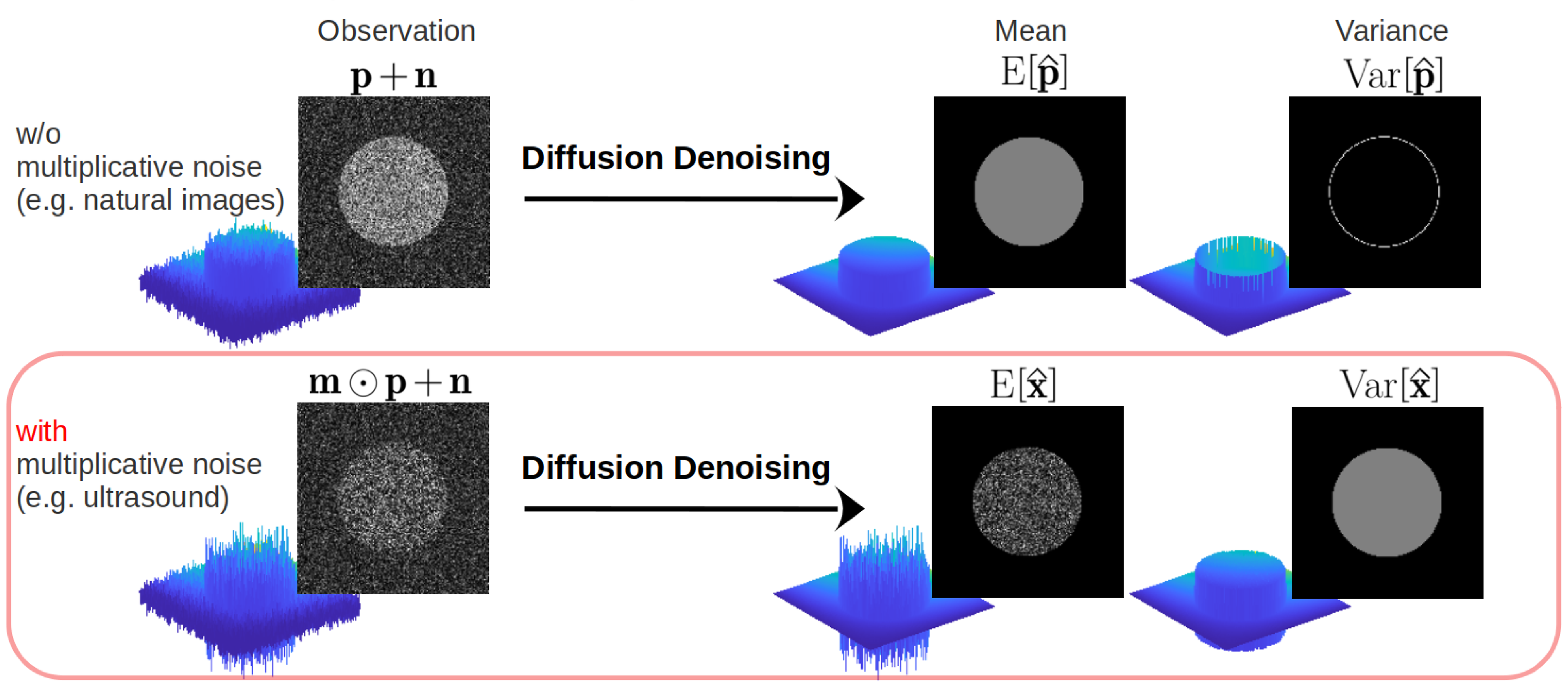


$$\mathbf{By} = \mathbf{BHx} + \mathbf{Bn}$$
$$\mathbf{By} = \mathbf{U\Sigma V}^\top\mathbf{x} + \mathbf{Bn}$$
$$\underbrace{\mathbf{\Sigma}^\dagger\mathbf{U}^\top\mathbf{By}}_{\text{noisy signal}} = \underbrace{\mathbf{V}^\top\mathbf{x}}_{\text{clean signal}} + \underbrace{\mathbf{\Sigma}^\dagger\mathbf{U}^\top\mathbf{Bn}}_{\text{noise}}$$

Assuming that $\mathbf{Bn} \sim \mathcal{N}(\mathbf{0}, \gamma^2\mathbf{I})$, each element in $\mathbf{\Sigma}^\dagger\mathbf{U}^\top\mathbf{Bn}$ is compared to the diffusion noise with variance σ_t^2 $(t = 1, \ldots, T)$:

$$\mathbf{\Sigma}^\dagger\mathbf{U}^\top\mathbf{Bn} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \frac{\gamma^2}{s_1^2} & & & & \\ & \ddots & & & \\ & & \frac{\gamma^2}{s_i^2} & & \\ & & & \ddots & \\ & & & & \frac{\gamma^2}{s_k^2} \end{bmatrix}\right) \quad \begin{cases} \leqslant \sigma_t^2, \text{ believe the measurements;} \\ > \sigma_t^2, \text{ rely on both the measurements and the diffusion prior;} \\ = +\infty, \text{ believe the diffusion prior.} \end{cases}$$

In a simple denoising scenario, multiple diffusion samples give different results depending on the absence or presence of multiplicative noise.



DRUSmean : reflectivity estimator, $\hat{\mathbf{x}}_{\text{DRUSmean}} = \frac{1}{C}\sum_{c=1}^{C}\hat{\mathbf{x}}_c$

DRUSvar : echogenicity estimator, $\hat{\mathbf{p}}_{\text{DRUSvar}} = \frac{1}{C-1}\sum_{c=1}^{C}|\hat{\mathbf{x}}_c - \hat{\mathbf{x}}_{\text{DRUSmean}}|^2$

4/ Experimental results

- Using a single plane wave (PW) with the delay-and-sum (DAS) method, calculated as $\mathbf{B}_1\mathbf{y}_1$, establishes the **baseline**.
- Employing 75 PWs with DAS, formulated as $\sum_{i=1}^{75}\mathbf{B}_i\mathbf{y}_i$, establishes the **gold standard**.

DENO[3] and DRUS[1] underwent 50 iteration steps for each sample, and each mean/variance image was constructed with $C = 10$ samples.

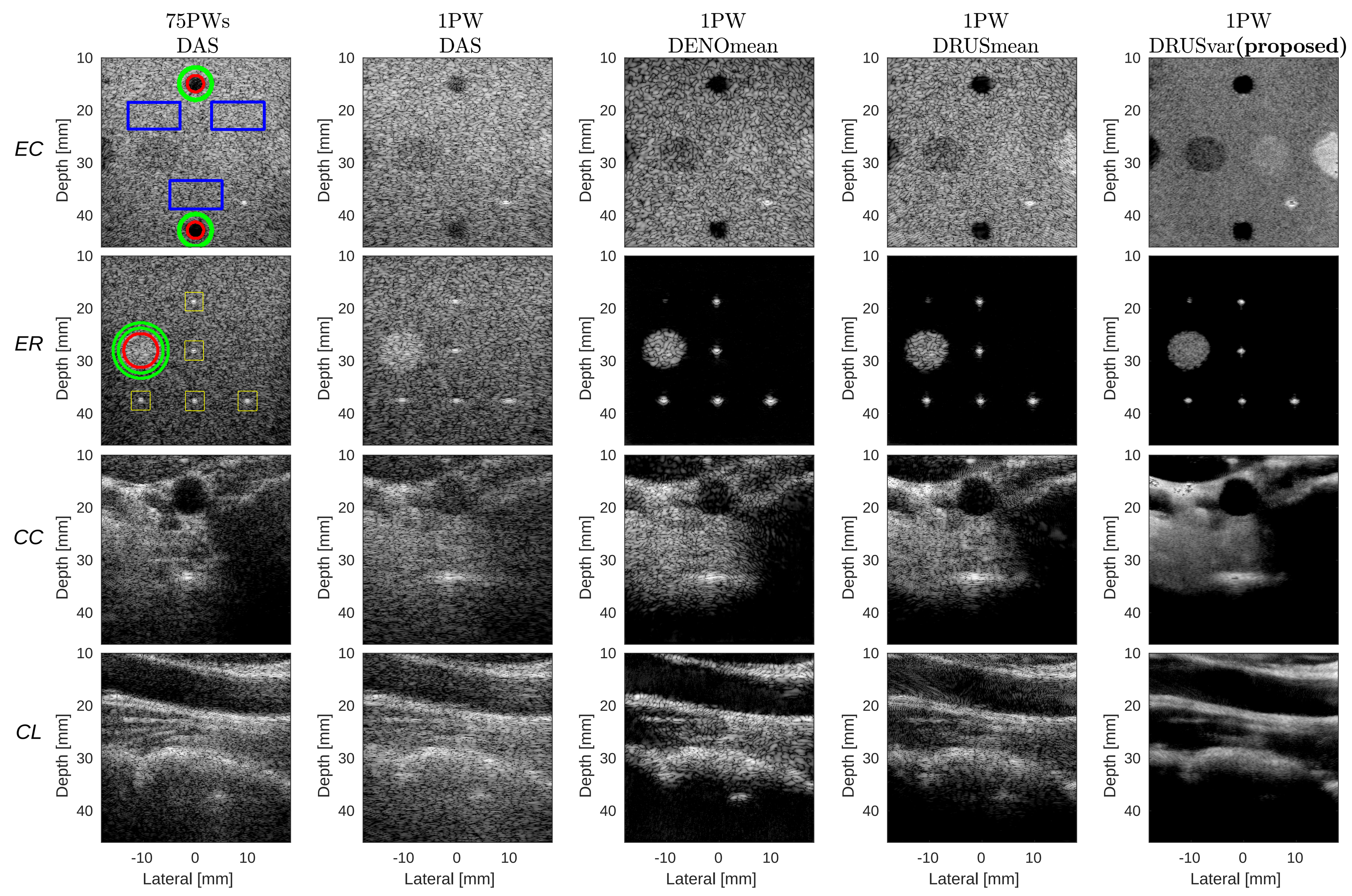


Figure 2 – Comparison of reconstructed images on the PICMUS datasets. All images are in decibels with a dynamic range [-60,0].

Table 1 – Quantitative comparison to SOTAs on the PICMUS phantom-based datasets. Best values bolded, second-best underlined.

		DAS (75PWs)	(1PW)	EMV (1PW)	MNV2 (1PW)	DNN-λ* (1PW)	DENOmean (1PW)	DRUSmean (1PW)	DRUSvar (1PW)
EC	gCNR↑	0.95	0.87	0.83	0.83	/	0.95	<u>0.97</u>	**0.98**
	SNR↑	1.92	<u>1.97</u>	/	/	/	1.93	1.87	**3.03**
ER	FWHM [mm] A↓	0.54	0.56	0.59	0.53	0.52	<u>0.31</u>	**0.24**	0.34
	FWHM [mm] L↓	0.56	0.87	<u>0.42</u>	0.77	0.52	0.64	0.54	**0.32**
	gCNR↑	0.77	0.69	/	/	/	<u>0.95</u>	<u>0.95</u>	**1.00**

5/ Comparison against a despeckling method

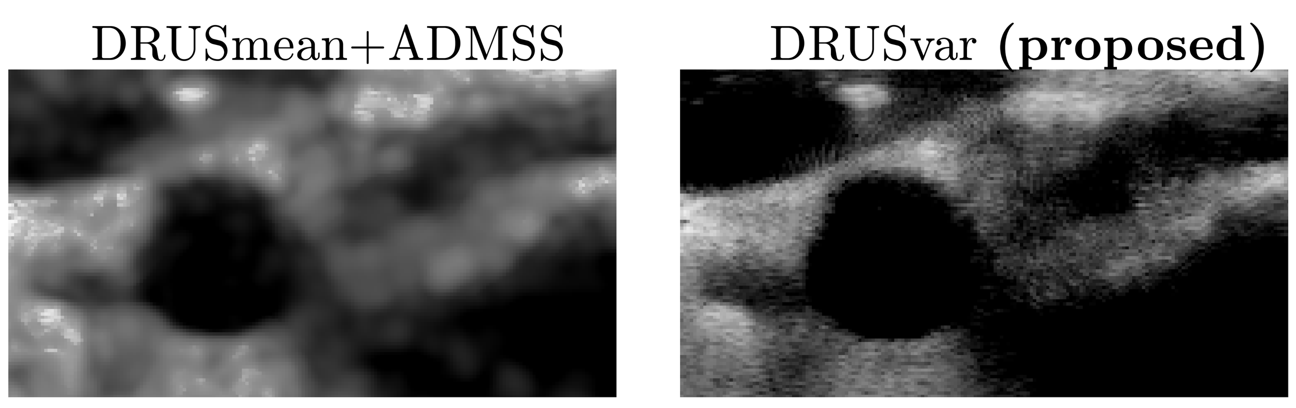


Figure 3 – Visual comparison of despeckled images on the *CC* dataset, in decibels [-60,0] dynamic range. ADMSS is an US despeckling method applied on beamformed images before log compression.

6/ Conclusion

Given the nature of multiplicative noise inherent to ultrasound, and the stochasticity of diffusion sampling, we explore a new application of DRUS [1] and introduce **DRUSvar** as an ultrasound echogenicity map estimator. We conduct experiments on real data, demonstrating the efficacy of the proposed variance imaging approach in achieving high-quality image reconstructions from single plane-wave acquisitions and in comparison to state-of-the-art methods.

References

[1] Y. Zhang, C. Huneau, J. Idier, and D. Mateus, "Ultrasound image reconstruction with denoising diffusion restoration models," in *Deep Generative Models*, 2024, pp. 193–203.

[2] B. Kawar, M. Elad, S. Ermon, and J. Song, "Denoising diffusion restoration models," *NeurIPS*, vol. 35, pp. 23 593–23 606, 2022.

[3] H. Asgariandehkordi, S. Goudarzi, A. Basarab, and H. Rivaz, "Deep ultrasound denoising using diffusion probabilistic models," in *IEEE IUS*, 2023.